Optimizing a Multi Period Deterministic Inventory Routing Problem in Agriculture Industries

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ABSTRACT

This paper develops a solving model for multi-period deterministic inventory routing problem (MP-DIRP) in supply chain focus on agriculture industries. The issue is how to improve the perishable product delivery process from a distribution centre (DC) to a collection of customers over a planning horizon. This paper proposed a model to achieve optimization in terms of costs respectively. The transportation problem is solved by using a simulation technique and developed using an algebraic modelling language which is known as a mathematical programming language (AMPL) to find an optimal solution within the period. The route for a distribution centre (DC) is set up by a supplier depending on the number of products assigned to each customer, which contributes to determining the best inventory routing problem (IRP) for each period. As a result, by applying the transportation model developed in this study, it is expected that the industries can achieve optimization by minimizing their total inventory and transportation costs.

Keywords: Supply chain; Multi-period inventory routing problem; Deterministic; Optimization

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1. INTRODUCTION

Logistics is one of the challenging areas in the supply chain system. The function of logistics in the company is to determine the amount of product and manage the demand wisely and accurately, consistency of delivery, and product availability (Udin et al., 2023). Companies should improve the supply chain system to remain competitive (Mohammad, 2019). Supply chain management (SCM) is a process to manage the product planning and services also included the process to transform the raw materials into a final product to deliver to the customers (Zha lechian et al., 2016). Furthermore, the supply chain also gives the essential element for the satisfaction of the customer in a growing number of product markets nowadays (Yadollahi et al., 2017).

Most of the companies are currently focusing on the possibilities of implementing the popular policy known as vendor managed inventory (VMI). The VMI policy is used to manage the inventories where the supplier manages all the activities for their inventory based on the customer's demand. The determination of the quantity is based on the available information of the inventory that helps to estimate accurately the customer requirements (Wu et al., 2021). This approach benefits both suppliers and customers. Other than that, the supplier can minimize the transportation costs by combining the multiple deliveries to optimize the capacity of the vehicle used. Implementing the VMI can improve the overall performance in the supply chain network (Mohammad, 2019).

Under the VMI policy, the inventory routing problem (IRP) is an approach used to determine the demand requirements and delivery process from suppliers to customers. IRP is a combination of two components, inventory management and vehicle routing planning which is challenging to solve (Iassinovskaia et al., 2017). IRP consists of a set of vehicles routes, the demand required, and delivery times that can minimize the total inventory and transportation costs. IRP mostly happens in the supply chain system where to manage the inventory and the transportation at the same time where the supplier takes the responsibility for the inventory replenishment to customers over a given planning horizon (Rahim et al., 2017).
Most of the researchers investigated the IRP by applying both existing and new methods for optimization algorithms. In its most basic form, a vehicle routing problem (VRP) is a routing problem involving a single warehouse and a collection of customers, with the goal of minimizing total cost while fulfilling the consumers. Due to the different optimization solutions and technologies used in solving the logistic problem, VRP continues to attract a lot of attention. Many logistics companies are trying to be the best at organizing product delivery by implementing the technology available today. However, not all implementations of VMI result in a positive outcome. Any difficulty and failure can happen because of the incorrect information data such as time and order from the customers and the supplier’s incompetence in decision-making decisions. Most of the researchers studied the IRP problem in which all customers’ demands is facing deterministic and constant rates over a planning period (Giroudeau et al., 2016; Guimarães et al., 2019; Bertazzi et al., 2019). Since the demand in real life is not always consistent or stochastic, this problem can be demonstrated as a multi-period stochastic unstationary demand rate. This research will discuss in detail the multi-period stochastic inventory routing problem (MP-SIRP) with unstationary demand rates.

2. LITERATURE REVIEW

Supply Chain Management (SCM) has become one of the most important policies for industry and keep growing in business since the previous three decades, either the concept or the way it approaches (Masteika & Cepinskis, 2015). The supply chain is a system that includes the supplier’s database, production facilities, distribution services, and customers linked by the customer information and product flow (Syafrudin et al., 2017). The SCM is important for the manufacturing sector but these days, it is now becoming more an important role in the food and fresh produce industries is care about the relationship created in a chain from suppliers to customers. The aim is to define the problems that exist between each of the units to achieve optimization which can minimize the total costs of production and maximize the company’s profit.

VMI is implemented to improve logistic services (Rohmer et al., 2019). From the previous researcher, the VMI is an alternative used by the customers in which the vendor is taking the position to manage and monitor the inventory of suppliers (Coelho et al., 2014; Rahimi et al., 2017; Bertazzi et al., 2019). A lot of problems can happen in the VMI. One of the problems is the inventory routing problem (IRP). To solve the IRP, a solution from a difficult mathematical problem must be developed. IRP consists of a combination of two problems which are inventory planning and vehicle routing (Rohmer et al., 2019). Vehicle routing problem (VRP) is the main role of distribution management. Many companies are involved in delivery and collection activities in doing business daily. The challenge is to develop a plan that consists of a trip from a warehouse, the customer with the demand is unknown as well as to estimate the minimum number of vehicles used to minimize the routing travelled costs. VRP is used to design the optimal routes for a vehicle to serve all the customers (Comert et al., 2018).

According to the earliest study by Bell et al. (1983), inventory planning and vehicle scheduling have a fluctuating adaptation of IRP. There are a lot of methodologies and algorithms in solving the IRP that have been proposed. For example, when customer demand is faced deterministic or stochastic, or when customers demand is expected deterministic or stochastic, a distinctive model can be developed (Rahim et al., 2014). Archetti et al (2007) were the first authors to propose a solution using the Branch-and-cut algorithms for a single product and single-vehicle IRP. They developed a specific formulation for optimization. This formulation was extended by Solyali & Sural (2011) where the replenishment strategy is proposed by studying the shortest route by applying a heuristic approach. Coelho & Laporte (2013) improved the method by introducing a branch-and-cut algorithm for solving multi-product multi-vehicle IRP with predictable consumer demand rates and no stock out. The author also applied a solution to improve the algorithm as the best solution. The aim is to get the approximate cost of a new solution while also improving the solution method.

Spliet & Desaulniers (2015) looked at a single supplier who produces a single product at each period of a finite horizon and distributes it to a collection of customers. They employ a homogeneous fleet of vehicles, and each customer has their own inventory capacity and initial stock. The author proposed a unique formulation to solve the IRP which is a state-of-the-art branch-price-and-cut algorithm. An advanced set of known rules and new families of finding inequalities, appending an adaption of the well-known amount of capacity inequalities to solve the column area subproblem. The computational results demonstrated that the suggested algorithm is valid, including valid inequalities, branching decisions, and other successful speed-up strategies.

Puga & Tancrez (2017) proposed a heuristic algorithm to solve the inventory problem with a dimension in a two-level supply chain which includes the warehouse, distribution centre, and customers. They also developed a model for a location for customers with uncertain demand rates. Larrain et al (2019) solve the multi-period IRP with due dates. Each customer is associated with a release date and a due date which is the product must be available at the supplier when the customer needs and when the customer must be visited.
3. METHODS

The multi-period inventory routing problem (MP-DIRP) involves a single distribution centre (DC) to distribute a product to a set of customers over a given planning horizon. The goal is to determine the appropriate demand quantities to be delivered to a set of customers, as well as the delivery time and vehicle delivery routes to supply the product to each customer, so that total inventory and transportation costs can be optimised over the planning horizon. The solving model used in this research is developed from a previous paper (Rahim et al., 2017, 2017b). We create a plan that incorporates DC as a supplier who will use a fleet of homogeneous trucks to distribute the product to a group of customers. To develop the MP-DIRP model, some assumptions must be made:

- Customer demand must be known in advance with the right amount.
- The vehicle’s capacity must be set enough to deliver the customer’s requirement.
- The cost of transportation must be proportional to vehicle travel times.
- Only a fleet of homogeneous vehicles is used to replenish the customers
- Split delivery is not allowed in the model

Let $H = \{1, 2, \ldots, T\}$ be a planning horizon set of planning periods denoted by $t$, and $H^+ = H \cup \{0\}$. The size in time units of the period is given by $\tau_t$, which the working hour is assumed to be eight hours. Let $S$ be the set of customers marked as $i$ and $j$, and $S^+ = S \cup \{r\}$, where $r$ represents the distribution centre (DC). The number of fleet vehicles ($V$) is used to serve all customers over the planning horizon. To develop the model, some relevant parameters and variables are used is presented below (Table 1 and 2).

Constraint (1) is the objective function that has four cost components (total vehicle’s fixed operating, total transportation cost, total delivery handling cost, and total inventory holding cost) at the distribution centre (DC) and customers. Constraints (2) ensure that the vehicle must visit each customer at once not more. Constraints (3) make sure that the vehicle must leave after it has been served and then go to the next customer or return to the DC. Constraints (4) ensure that vehicles complete their routes within one travel period so that the total vehicle’s travelling time should not exceed the total working hours. Constraints (5) estimate the customer’s quantity required to be delivered. Constraint (6) assure that the quantity carried cannot exceed the maximum loading capacity of the vehicle. The product balance equation at the customers in constraint (7). To indicate the final level of product at customer $j$ at the end of the period is of the same magnitude as its initial product shows in constraints (8). Constraints (9) ensure that a vehicle cannot be used to serve any customer only if the customers are selected. Constraint (10) is the entirety and sign constraint to be imposed on the variables.

### Table 1: Parameters

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_j$</td>
<td>Fixed handling cost per delivery (in RM) at location $j \in S^+$ (customers and DC)</td>
</tr>
<tr>
<td>$n_j$</td>
<td>Product holding cost per period at location $j \in S^+$ (in RM/kg)</td>
</tr>
<tr>
<td>$\psi^v$</td>
<td>Vehicle's operating fixed cost $v \in V$ (in RM/kg)</td>
</tr>
<tr>
<td>$\delta_v$</td>
<td>Vehicle’s travelling cost $v \in V$ (in RM/vehicle)</td>
</tr>
<tr>
<td>$k^v$</td>
<td>Vehicle’s capacity $v \in V$ (in kg)</td>
</tr>
<tr>
<td>$v_a$</td>
<td>Vehicle’s average speed $v \in V$ (in km/hour)</td>
</tr>
<tr>
<td>$\theta_{ij}$</td>
<td>The duration trip from customer $i \in S^+$ to customer $j \in S^+$ (in an hour)</td>
</tr>
<tr>
<td>$d_j$</td>
<td>The constant demand rate at customer $j$ (in kg/hour).</td>
</tr>
<tr>
<td>$l_{j0}$</td>
<td>The levels of the initial product (in kg) at each customer $j \in S$</td>
</tr>
</tbody>
</table>

### Table 2: Variables

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{ij}$</td>
<td>The product quantity remaining in the vehicle (in kg) $v \in V$ when the vehicle travels directly to location $j \in S^+$ from location $i \in S^+$. The quantity will become zero (0) when the trip $(i, j)$ is not having a tour by the vehicle $v \in V$</td>
</tr>
<tr>
<td>$q_j$</td>
<td>The delivery quantity (in kg) to the location $j \in S$, and 0</td>
</tr>
<tr>
<td>$l_j$</td>
<td>The level of the product at the location (customers and DC) $j \in S^+$</td>
</tr>
<tr>
<td>$x_{ij}^r$</td>
<td>If location $j \in S^+$ is visited immediately after location $i \in S^+$ by vehicle $v \in V$, and 0 (A binary variable set to 1)</td>
</tr>
<tr>
<td>$y^v$</td>
<td>If vehicle $v \in V$, and 0 (A binary variable set to 1)</td>
</tr>
</tbody>
</table>
\[(MP-\text{DIRP}) \text{minimize} \]

\[CV = \sum_{v \in V} \left[ \psi^v y^v + \sum_{w \in S} \sum_{j \in S^+} (\delta^v \psi^v + \varphi^v_j) x^v_{w,j} \right] + \sum \eta_j l_j \quad (1)\]

Subject to:

\[\sum_{j \in S^+} x^v_{w,j} \leq 1, \quad \forall j \in S \quad (2)\]

\[\sum_{j \in S^+} x^v_{w,j} - \sum_{j \in S} x^v_{w,j} = 0, \quad \forall j \in S^+, v = V \quad (3)\]

\[\sum_{j \in S} \sum_{j \in S^+} \theta^v_j x^v_{w,j} \leq \tau_t, \quad v = V \quad (4)\]

\[\sum_{v \in V} \sum_{i \in S} Q^v_{i,j} - \sum_{v \in V} \sum_{i \in S} Q^v_{i,j} = q_{jt}, \quad \forall j \in S \quad (5)\]

\[Q^v_{i,j} \leq k^v x^v_{i,j}, \quad \forall i, j \in S^+, v \in V \quad (6)\]

\[l_{j-1} + q_r - l_r = d_j \tau_t, \quad \forall j \in S \quad (7)\]

\[l_{j_0} \leq l_j, \quad \forall j \in S \quad (8)\]

\[x^v_{i,j} \leq y^v, \quad \forall j \in S, v \in V \quad (9)\]

\[x^v_{i,j}, y^v \in \{0, 1\}, l_{j_0}, l_j \geq 0, q_r \geq 0, \quad \forall i, j \in S^+, v \in V \quad (10)\]

4. RESULTS AND DISCUSSION

4.1 Mathematical Model Validation

Table 3 shows the coordinates for the customers which are denoted as \((x, y)\) with the demand rates are assumed to be deterministic. There are 10 customers sketched around the distribution centre (DC) in a square of 30 by 30 km, with average demand rates are known in advance between 0.1 to 3 tons per hour. The vehicle loading capacity, \(k^v = 40\) tons for a fleet homogeneous vehicle, \(V\). The operating fixed cost for the vehicle, \(\psi^v = RM200\) per vehicle. The average speed for the vehicle is 60 km per hour and the travel cost, \(\delta^v = RM4\) per kilometre. The handling fixed cost per delivery, \(\varphi_{jt} = RM100\) for all customers and considered same with the time units, \(\tau_t = 8\) hours. Figure 1 illustrates the location for DC and customers based on the coordinates (Table 1). On the map given, the customers are located around the DC with the demand rate are deterministic and generated randomly between 0 to 6 kg per hour. Each customer's product is replenished by a fleet of homogeneous vehicles. We assumed that the vehicle speed is up to 50 km per hour.

<table>
<thead>
<tr>
<th>Customers</th>
<th>Demand Rate (ton/hour)</th>
<th>Quantity Delivery (Qdev)</th>
<th>Delivery cost (RM)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(T_1)</td>
<td>(T_2)</td>
<td>(T_3)</td>
</tr>
<tr>
<td>1</td>
<td>1.83</td>
<td>2.00</td>
<td>2.25</td>
</tr>
<tr>
<td>2</td>
<td>2.04</td>
<td>2.10</td>
<td>2.85</td>
</tr>
<tr>
<td>3</td>
<td>2.55</td>
<td>2.00</td>
<td>2.73</td>
</tr>
<tr>
<td>4</td>
<td>1.35</td>
<td>2.93</td>
<td>1.62</td>
</tr>
<tr>
<td>5</td>
<td>1.62</td>
<td>1.22</td>
<td>2.20</td>
</tr>
<tr>
<td>6</td>
<td>2.51</td>
<td>2.05</td>
<td>1.93</td>
</tr>
<tr>
<td>7</td>
<td>1.41</td>
<td>1.12</td>
<td>1.20</td>
</tr>
<tr>
<td>8</td>
<td>2.25</td>
<td>2.96</td>
<td>2.90</td>
</tr>
<tr>
<td>9</td>
<td>1.52</td>
<td>2.10</td>
<td>2.65</td>
</tr>
<tr>
<td>10</td>
<td>1.82</td>
<td>1.38</td>
<td>2.15</td>
</tr>
</tbody>
</table>
4.2 MP-DIRP Simulations

Next, we solve the MP-DIRP by using a mathematical programming language (AMPL) with CPLEX 12.10.0.0. After that, implementing the optimal solution shows that the result where only one vehicle is allowed to be used to replenish all 10 customers over a given planning horizon.

- Period 1: $t = 1$, Vehicle: $V = V_1$: \{(1,10,4), (3,5), (6,2), (8), (9,7)\}
- Period 2: $t = 2$, Vehicle: $V = V_1$: \{(2,6), (5,3), (8,1), (9,7), (10,4)\}
- Period 3: $t = 3$, Vehicle: $V = V_1$: \{(1,10), (2,6), (3,5), (8,4), (9,7)\}

Figure 2 below illustrated the results of the optimal solution for MP-DIRP. As we can see that the vehicle in period $t = 1$ is denoted as $V_1$, creates four multi-travelling routes of the solution by using a single vehicle to replenish the inventory \{(1,10,4), (3,5), (6,2), (8), (9,7)\}. Then, in period $t = 2$ within the 8 hours, the vehicle creates four multi-routes and one direct shipping to replenish the inventory where the vehicle takes \{(2,6), (5,3), (8,1), (9,7), (10,4)\}. Next, for the $t = 3$, the vehicle makes five multi-routes which are \{(1,10), (2,6), (3,5), (8,4), (9,7)\} to replenish the inventories before returning to the distribution centre (DC) with the optimum cost is RM4716.00. In conclusion, the multi-tour distribution routes show that the vehicle capacity is being optimized within the period to replenish the inventory to the customers. The solution for the three-consecutive period (3 periods) of MP-DIRP is illustrated in Figure 2. As a result, the change in the capacity of the vehicle would affect the cost to replenish the product. In addition, we simulate the example of using 100 kg and 200 kg of vehicle capacity to compare the effect that can happen. It shows that the decrease in total cost from RM1498.00 (100 kg) to RM1412.00 (200 kg) which is cheaper. Therefore, we can estimate that the bigger the capacity of the vehicle, the smaller route will be taken to deliver the product to customers.

4.3 Additional Instance For MP-DIRP

Table 4 above shows the instance of changes in vehicle capacity. We develop the data where the location of the DC and the customers are different respectively. According to Table 4, the instance is denoted as an (R10-0-T1) which R10 refers to the 10 customers, 0 is the number of instances and T1 refers to the time which is only one period. As we can see, the increase in vehicle capacity affects the cost to become lower compared with the small capacity. By using a bigger capacity, the vehicle can deliver the product to the customers directly without returning to the DC often. In that case, we can understand that adding the vehicle capacity can reduce the route taken and automatically reduce the transportation cost-effectively.

![Figure 1: Normal distribution without optimization](https://www.compendiumpaperasia.com)
a) Period 1: $t = 1$, Vehicle: $V = V_1$: \{(1,10,4), (3,5), (6,2), (8), (9,7)\}

b) Period 2: $t = 2$, Vehicle: $V = V_1$: \{(2,6), (5,3), (8,1), (9,7), (10,4)\}

c) Period 3: $t = 3$, Vehicle: $V = V_1$: \{(1,10), (2,6), (3,5), (8,4), (9,7)\}

Figure 2: The optimal solution of the 10-customers (MP-DIRP)
5. CONCLUSIONS

In conclusion, this research provides a mathematical programming model for multi-period deterministic inventory routing problems (MP-DIRP) regarding the ideal amounts required to be delivered to the customers, vehicle delivery routes, and delivery time. So, this model solving is suitable to be used to minimize the total inventory and transportation cost in IRP while performing some level of services at every customer during the planning period of the planning horizon. The proposed model recognizes the inventory management strength and weaknesses, which can give a positive impact on the decision-making process for the company to achieve the minimization of the total inventory and transportation costs.

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